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Also solved by E. B. Escott, and G. B. M. Zerr. Professor Escott solved the problem by putting the general term equal to $A+Bn+Cn(n+1)+...+Gn(n+1)(n+2)(n+3)(n+4)(n+5)$. Then by letting $n=0$, -1, -2, etc., he determines A, B, ..., G. The general term is thus reduced to five terms of the form $n(n+1)...(n+r-1)$. Since the sum of a series whose general term is $n(n+1)(n+2)...(n+r-1)$ is $[n(n+1)...(n+r-1)]/[r+1]$ finds the sum which agrees with that obtained by Mr. DeLand.

Dr. Zerr decomposed the general term in a similar way and after summing the five similar series thus arising he gets the same result as that given above.

GEOMETRY.

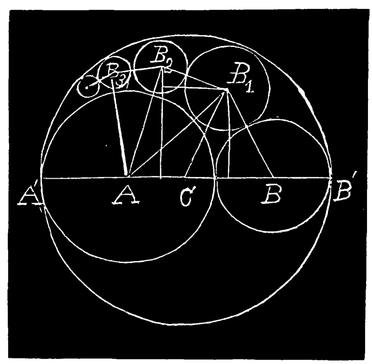
326. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

The circle C of radius pR encloses the circles A_1, B_1 of radii R and $(p-1)R$, respectively; the circle B_1 is tangent to A_1, B_1, C_1; the circle B_2 is tangent to A, B_1, C; the circle B_3 to A, B_2, C, ..., B_n to A, B_{n-1}, C. Find the radius of the circle B_n.

Solution by the PROPOSER.

First find the locus of centers of circles tangent to A and C, taking A' the point of contact of A and C as the origin.

Let r, r_1, r_2, ..., r_n be the radii of B, B_1, ..., B_n, respectively; r'=radius of any circle tangent to circles whose centers are A, C; and x, y co-ordinates of its centers. Then $(r'+R)^2-(R-x')^2=(pR-r')^2-(pR-x')^2=y'^2$...(1).



$$\therefore r=\frac{(p-1)x'}{p+1}\text{...(2), and } x'=\frac{(p+1)r'}{p-1}\text{...(3).}$$

Substituting the value of x' in (1), we have

$$(R+r')^2-(R-\frac{p+1}{p-1}r')^2=y'^2.$$

$\therefore y'=\dfrac{2}{p-1}\sqrt{[p(p-1)Rr'-pr'^2]}$. Since $r=(p-1)R$ and $x=R(p+1)$, $(p-1)R$ is of the form $\dfrac{p(p-1)R}{0^2(p-1)^2+p}$.

2. Find r_1. Join centers of A, B, and C with B_1, B_2, ..., B_n. Draw perpendiculars from centers B_1, B_2, ..., to the diameter of C passing through A'.

$$(r+r'_1)^2-(x-x_1)^2=(pR-r_1)^2-(pR-x_1)^2\text{...(4).}\quad x_1=\left(\frac{p+1}{p-1}\right)r_1.$$

Substitute the values of x, r, x_1 in (4); whence

$$4Rr_1(p^2-p+1)=(p-1)pR^2.\quad \therefore r_1=\frac{p(p-1)R}{(p-1)^2+p}=\frac{p(p-1)R}{1^2(p-1)^2+p}.$$

(3). Find r_2. Let $(p-1)=s$. $(r_1+r_2)^2-(x_1-x_2)^2=(y_1-y_2)^2$..(5).

$$r_1^2=\left(\frac{ps}{s^2+p}\right)^2 R^2;\ x_1^2=\frac{p^2(p+1)^2R^2}{(s^2+p)^2};\ x_2^2=\frac{(p+1)^2}{s^2}r_2^2;\ y_1=\frac{2pRs^2}{s(s^2+p)};$$

$$y_2=\frac{2}{s}\sqrt{[pRsr_2-pr_2^2]}.$$

Equation (5) becomes, after taking square root of both members,

$$\sqrt{\frac{(ps)^2R^2}{s^2+p}-\frac{p^2(p+1)^2R^2}{(s^2+p)^2}+\frac{2p(p+1)^2r_2R}{s(s^2+p)}+r_2^2-\frac{(p+1)^2r_2^2}{s^2}}$$

$$=\frac{2pRs^2}{s(s^2+p)}-\frac{2}{s}\sqrt{[pRsr_2-pr_2^2]}\ldots(6).$$

After clearing of radicals and reducing, (6) becomes

$$\frac{p[(s^2-p^2)+4s^2p]R^2}{(s^2+p)^2}+\frac{p+4s^2}{s^2}r_2^2-\frac{2[s^2(2p^2-p+2)+p^2]Rr_2}{s(s^2+p)}=0\ldots(7).$$

$$\therefore r_2=\frac{s[s^2(2p^2-p+2)+p^2]R}{(s^2+p)(p+4s^2)}$$

$$-\frac{R\sqrt{\{[s^2(2p^2-p+2)+p^2]^2-p(p+4s^2)(s^2+p)^2\}}}{(s^2+p)(p+4s^2)},$$

$$R\sqrt{\{[s^2(2p^2-p+2)+p^2]^2-p(p+4s^2)(s^2+p)^2\}}$$
$$=2s^2(s^2+p)=s^2(2p^2-2p+2).$$

$$\therefore r_2=\frac{Rsp(s^2+p)}{(s^2+p)(p+4s^2)}=\frac{p(p-1)R}{2^2(p-1)^2+p}.$$

3. Find r_n. Assume that $r_{n-1}=\dfrac{p(p-1)R}{(n-1)^2\,(p-1)^2+p}$.

Let $(n-1)=t$; then $r_{n-1}=\dfrac{pRs}{t^2s^2+p}$. $(r_t+r_n)^2-(x_t-x_n)^2=(y_t-y_n)^2\ldots(8)$.

$$x_t=\frac{p(p+1)R}{t^2s^2+p};\ y_t=\frac{2tpsR}{t^2\,s^2+p};\ y_n=\frac{2}{s}\sqrt{[psRr_n-pr_n^2]}.$$

As in case 2, (8) becomes

$$\frac{[t^2s^2+p-(p^2+1)]^2+4t^2ps^2}{(s^2t^2+p)^2}r_n{}^2-\frac{2ps\,[(t^2s^2-p)t^2s^2+p-(p^2+1)]Rr_n}{s^2t^2+p}$$

$$+\frac{4t^2(p^2+1)s^2ps}{s^2t^2+p}Rr_n+\frac{p^2s^2[\,(t^2s^2-p)^2+4t^2ps^2]R^2}{(s^2t^2+p)^2}=0\ldots(9);\text{ whence}$$

$$r_n=\frac{psR\{(t^2s^2-p)\,[t^2s^2+p-(p^2+1)]+2t^2s^2(p^2+1)\}}{(t^2s^2+p)\,[t^2s^2+p-(p^2+1)^2+4t^2ps^2}$$

$$-[pRs\surd\{(t^2s^2-p[t^2s^2+p-(p^2+1)+2t^2s^2(p^2+1)]^2$$
$$-(t^2s^2+p)^2[t^2s^2+p-(p^2+1)]^2+4pt^2s^2)\}]$$
$$/(t^2s^2+p)\,[t^2s^2+p-(p^2+1)]^2+4t^2ps.$$

The quantity under the radical$=2t^3s^4+2tps^2$.

$$\therefore r_n=\frac{pRs\{[(t^2s^2-p)\,(t^2s^2+p-(p^2+1)]+2t^2s^2(p^2+1)-2ts^2(s^2t^2+p)\}}{(t^2s^2+p)\,[t^2s^2+p-(p^2+1)]^2+4ps^2t^2}$$

$$=[pRs\{(t^2s^2-p)\,[(t^2s^2+p-(p^2+1)]+2t^2s^2(p^2+1)]-2ts^2(s^2t^2+p)\}]$$
$$/\{(t^2s^2-p)\,[(t^2s^2+p-(p^2+1)]+2t^2s^2(p^2+1)$$
$$-2ts^2(s^2t^2+p)\}\{(t+1)^2(p-1)^2+p\}.$$

$$=\frac{psR}{(t+1)^2(p-1)^2+p}=\frac{p(p-1)R}{n^2(p-1)^2+p}.$$

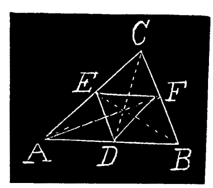
Since it has been shown that this expression is true for B_1 and B_2, it follows that it is true for B_n.

Excellent demonstrations were received from G. B. M. Zerr and C. E. White.

327. Proposed by J. C. CORBIN, Pine Bluff, Ark.

In triangle ABC, the triangle DEF is formed by joining the feet of the medians and four parallelograms are also formed, viz., $AEFD$, $BFED$, and $CEDF$. Let a, b, c; d, e, f represent the three medians of ABC, and the three sides of DEF. Then the sum of the squares of the six diagonals equals the sum of the squares of the twelve sides of the parallelograms, which are equal in sets of four. That is, $a^2+b^2+c^2+d^2+e^2+f^2=4(d^2+e^2+f^2)$, or $a^2+b^2+c^2=3(d^2+e^2+f^2)=3/4(AB^2+BC^2+CA^2)$.

Solution by J. H. MEYER, S. J., Augusta, Ga.



Let $CD=a$, $AF=b$, $EB=c$, $DF=e$, $EF=d$, and $ED=f$. Now, by geometry, we know that

a^2+d^2 in parallelogram $ECFD=2f^2+2a^2$;
b^2+f^2 in parallelogram $AEFD=2e^2+2d^2$;
c^2+e^2 in parallelogram $BFED=2f^2+2d^2$.